



# Different Decoders

• For naïve decoder,  $\hat{x}_{\text{naïve}}(y) = y$ . Therefore, we can simply copy the values in the  $y$ -column into its decoding table.

• For DIY decoder, the decoding table is provided. (Alternatively, some equation may be given; in which case, we can simply plug-in each of the possible  $y$  values to get  $\hat{x}_{\text{DIY}}(y)$ .)

• Deriving the MAP and ML decoder follows almost the same recipe: select the max value in each column and read the corresponding  $x$ -value. The difference is that the MAP decoder uses the  $\mathbf{P}$  matrix but the ML decoder uses the  $\mathbf{Q}$  matrix.

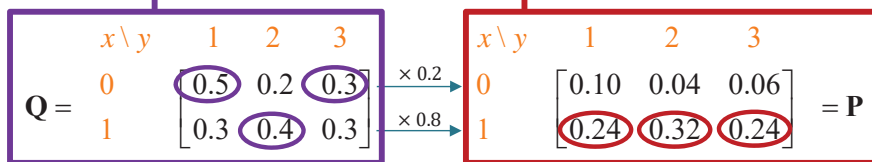
$y$	$\hat{x}_{\text{naïve}}(y)$	$\hat{x}_{\text{DIY}}(y)$	$\hat{x}_{\text{MAP}}(y)$	$\hat{x}_{\text{ML}}(y)$
1	1	1	1	0
2	2	1	1	1
3	3	0	1	0

[Ex. 3.25]

[Ex. 3.27]

[Ex. 3.35]

[Ex. 3.46]



## Finding $P(\mathcal{E})$

- Once a decoder  $\hat{x}(y)$  is defined, we can find its corresponding  $P(\mathcal{E})$  easily from the  $\mathbf{P}$  matrix:
  - Write  $\hat{x}(y)$  values on top of the  $y$  values for the  $\mathbf{P}$  matrix.
  - For each column  $y$  in the  $\mathbf{P}$  matrix, circle the element whose corresponding  $x$  value is the same as  $\hat{x}(y)$ .

$y$	$\hat{x}_{\text{naïve}}(y)$	$\hat{x}_{\text{DIY}}(y)$	$\hat{x}_{\text{MAP}}(y)$	$\hat{x}_{\text{ML}}(y)$
1	1	1	1	0
2	2	1	1	1
3	3	0	1	0
$P(\mathcal{E})$	0.76	0.38	0.20	0.52

- $P(\mathcal{C})$  = the sum of the circled probabilities.
- $P(\mathcal{E}) = 1 - P(\mathcal{C})$ .

$\hat{x}_{\text{naïve}}(y)$	1	2	3	$\hat{x}_{\text{DIY}}(y)$	1	1	0	$\hat{x}_{\text{MAP}}(y)$	1	1	1	$\hat{x}_{\text{ML}}(y)$	0	1	0
$x \setminus y$	1	2	3	$x \setminus y$	1	2	3	$x \setminus y$	1	2	3	$x \setminus y$	1	2	3
0	0.10	0.04	0.06	0	0.10	0.04	0.06	0	0.10	0.04	0.06	0	0.10	0.04	0.06
1	0.24	0.32	0.24	1	0.24	0.32	0.24	1	0.24	0.32	0.24	1	0.24	0.32	0.24